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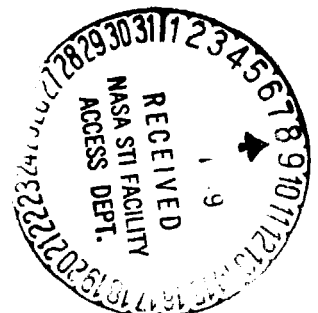
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Prepared for

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FLUID DYNAMIC PROBLEMS ASSOCIATED
WITH AIR-BREATHING PROPULSIVE SYSTEMS

UNIVERSITY OF ILLINOIS
AT URBANA-CHAMPAIGN
URBANA, ILLINOIS 61801
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CALCULATIONS OF TRANSONIC BOATTAIL FLOW AT A
SMALL ANGLE OF ATTACK

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FOREWORD

This research was carried out under the partial financial support of NASA through Research Grant NGL 14-005-140 entitled "Fluid Dynamic Problems associated with Air-Breathing Propulsive Systems." Mr. B. H. Anderson and Dr. L. J. Bober of the Wind Tunnel and Flight Test Division, Lewis Research Center, NASA, served as technical monitors of this research grant. This report deals with the problem of a transonic flow past a boattailed afterbody under a small angle of attack.

ABSTRACT

A transonic flow past a boattailed afterbody under a small angle of attack has been examined. It is known that the viscous effect offers significant modifications of the pressure distribution on the afterbody. Thus, the formulation for the inviscid flow is based on the consideration of a flow past a non-axisymmetric body. The full three-dimensional potential equation is solved through numerical relaxation, and quasi-axisymmetric boundary layer calculations are performed to estimate the displacement effect. It has been observed again that the viscous effects are not negligible. The trend of the final results agreed well with the experimental data. It is believed that the general three-dimensional turbulent boundary layer calculations must be employed before excellent agreement with the experimental data can be achieved.

In the attempt to evaluate the performance of an air-breathing propulsive system of a supersonic aircraft operating at subsonic cruise conditions, an effort has been made to calculate the flow field associated with a transonic flow past a boattailed afterbody [1,2]* at zero angle of attack. Since this afterbody is always immersed in a relatively thick boundary layer, built-up along the body from the leading edge, it has been found that the pressure distribution on the boattailed afterbody is always strongly influenced by the existence of the viscous layer. It has been also learned that the small disturbance potential equation is not adequate under this situation to calculate the corresponding inviscid flow within the transonic flow regime. Instead, the full potential equation must be used and the influence of the viscous layer must be accounted for by correcting the inviscid flow configuration through the displacement considerations. The pressure distribution on the body produced from these calculations showed excellent agreement with the experimental data when the flow has not been separated and good qualitative indication of the flow when it is separated from the rear portion of the afterbody. This series of investigations also fully illustrated the importance of the interaction between the viscous layer and the inviscid flow.

The present effort, a sequel to this series of investigations, extends these considerations into the situation with small angle of attack. The same axisymmetric model

*Numbers in brackets refer to entries in REFERENCES.

with boattailed afterbody [1,2,3] is adopted for this study whose geometric configuration is shown in Fig. 1. It should be noted that this extension leads immediately into considerable trouble and difficulty. For an axisymmetric body, cases with angle of attack are of the three-dimensional nature since the flow field also varies in the meridional direction. The computational effort (computer time) multiplies with the number of meridional planes under consideration. In addition, the boundary layer growth along the body will also be of a three-dimensional nature, and the corrections to the inviscid body configuration would lead itself into a non-axisymmetric one even if the solid body is of the axisymmetric shape initially.

For an axisymmetric body, the coordinates to describe such a flow problem are naturally of the cylindrical system. Due to the flow with angle of attack, the radial velocity component along the axis will no longer vanish and some of the terms in the general three-dimensional potential equation in this coordinate system will approach infinity.* It is thus obvious that this equation cannot be employed for the flow along the axis and additional manipulations are needed to overcome this problem.

Since the compressible turbulent three-dimensional boundary layer calculations have not been developed, it is assumed that under the condition of small angle of attack,

*It is, nevertheless, interesting to observe that the uniform approaching flow condition with angle of attack satisfies this mathematical expression describing the potential equation in the cylindrical system of coordinates.

the viscous portion of the cross flow may not be very important, and the boundary layer growth can be approximated by a quasi-axisymmetric viscous flow calculations. Thus, the main purpose of the present effort is to study the feasibility of carrying out the three-dimensional transonic flow calculations, and developing the computer program for its inviscid flow. It is felt that it is simple to replace in the future the quasi-axisymmetric viscous flow treatment by a full three-dimensional compressible turbulent boundary layer calculation when it becomes available. Indeed, it has been found from the results of the present investigation that the equivalent inviscid configuration of the aforementioned boattailed afterbody is sensitive to the influence of the viscous flow, and accurate results can only be produced from the three-dimensional turbulent boundary layer considerations. The detailed treatment of this problem in its present form is described in the following sections.

BASIC GOVERNING EQUATIONS

Inviscid Flow

Referring to Fig. 2 where an axisymmetric body at an angle of attack is depicted, the compressible three-dimensional potential equation in the cylindrical system of coordinates (z, r, ω) governing the flow is given by

$$\begin{aligned}
 & [1 - (\phi_z^2/c^2)] \phi_{zz} + [1 - (\phi_r^2/c^2)] \phi_{rr} + [1 - (\phi_\omega^2/r^2 c^2)] \\
 & \cdot (\phi_{\omega\omega}/r^2) - [(2\phi_r \phi_\omega)/(r^2 c^2)] \phi_{r\omega} \\
 & - [(2\phi_\omega \phi_z)/(r^2 c^2)] \phi_{\omega z} - [(2\phi_r \phi_z)/c^2] \\
 & \cdot \phi_{rz} + (\phi_r/r) [1 + (\phi_\omega^2/r^2 c^2)] = 0 \quad (1)
 \end{aligned}$$

where the subscripts denote partial differentiations and c denotes the local speed of sound which is related to the stagnation velocity of sound c_0 and the free stream flow condition through

$$c^2 = c_0^2 - [(\gamma - 1)/2] [\phi_z^2 + \phi_r^2 + \phi_\omega^2/r^2] \quad (2)$$

with $c_0^2 = c_\infty^2 + [(\gamma - 1)/2] v_\infty^2$. Upon introducing

$\phi = v_\infty (z \cos \alpha + r \cos \omega \sin \alpha + \phi)$, the disturbed potential ϕ is found to satisfy

$$\begin{aligned}
 & [1 - (U^2/C^2)] \phi_{zz} + [1 - (V^2/C^2)] \phi_{rr} + [1 - (W^2/C^2)] \\
 & \cdot [(\phi_{\omega\omega} - r \cos \omega \sin \alpha)/r^2] - (2VW/C^2) \\
 & \cdot [(\phi_{r\omega} - \sin \omega \sin \alpha)/r] - (2UW/C^2) (\phi_{\omega z}/r) \\
 & - (2UV/C^2) \phi_{zr} + (V/r) [1 + (W^2/C^2)] = 0 \quad (3)
 \end{aligned}$$

where

$$\begin{aligned} U &= \phi_z/V_\infty = \cos \alpha + \phi_z; \\ V &= \phi_r/V_\infty = \cos \omega \sin \alpha + \phi_r; \text{ and} \\ W &= \phi_\omega/r V_\infty = \phi_\omega/r - \sin \omega \sin \alpha \end{aligned} \quad (4)$$

and

$$C^2 = (c^2/V_\infty^2) = 1/M_\infty^2 + [(\gamma - 1)/2] (1 - U^2 - V^2 - W^2)$$

In anticipating that the viscous effect will offer important corrections to the configuration of the body, and this correction varies in the meridional direction, one thus expects that the effective body is non-axisymmetric, namely, r_b is a function of both z and ω . One may introduce transformations according to

$$\xi = z; \quad \eta = r - r_b(z, \omega); \text{ and} \quad \Omega = \omega \quad (5)$$

and an additional transformation of the form

$$\zeta = [B\eta/(1 + B\eta)] \quad (6)$$

so that the condition of $\eta \rightarrow \infty$ is equivalent to $\zeta = 1$ in the finite computational space. B ($B = 8$) is a stretch parameter. Equation (3) appears now in the new system of coordinates (ξ, ζ, Ω) as

$$\begin{aligned} DD\phi_{\xi\xi} + EE \cdot B^2(1 - \zeta)^4 \phi_{\zeta\zeta} + FF\phi_{\Omega\Omega} \\ + B(1 - \zeta)^2 \cdot DE\phi_{\xi\zeta} + B(1 - \zeta)^2 \cdot EF\phi_{\zeta\Omega} + FD\phi_{\Omega\xi} \\ + B(1 - \zeta)^2 [E - 2B(1 - \zeta) \cdot EE] \phi_\zeta + G = 0 \end{aligned} \quad (7)$$

where

$$DD = [1 - (U^2/C^2)] \quad (8a)$$

$$\begin{aligned} EE &= [1 - (U^2/C^2)] r_{bz}^2 + [1 - (V^2/C^2)] \\ &+ [1 - (W^2/C^2)] (r_{b\omega}^2/r^2) + [(2VW r_{b\omega}/(r C^2))] \\ &- [(2WU r_{bz} r_{b\omega})/(r C^2)] \\ &+ (2UV/C^2) r_{bz} \end{aligned} \quad (8b)$$

$$FF = [1 - (W^2/C^2)]/r^2 \quad (8c)$$

$$DE = [2WU/(r C^2)] r_{b\omega} - 2[1 - (U^2/C^2)] r_{bz} \\ - (2UV/C^2) \quad (8d)$$

$$EF = [2UW/(r C^2)] r_{bz} - [2VW/(r C^2)] \\ - [2(1 - W^2/C^2) r_{b\omega}]/r^2 \quad (8e)$$

$$FD = - [2WU/(r C^2)] \quad (8f)$$

$$E = [2UW/(r C^2)] r_{b\omega z} - [1 - (U^2/C^2)] r_{bzz} \\ - [1 - (W^2/C^2)] (r_{b\omega\omega}/r^2) \quad (8g)$$

$$G = [1 + (W^2/C^2)] (V/r) + [2VW/(r C^2)] \sin \omega \sin \alpha \\ - [1 - (W^2/C^2)] [(\cos \omega \sin \alpha)/r] \quad (8h)$$

with $r = r_b + \zeta/[B(1 - \zeta)]$ and the subscripts z, ω for r_b indicate respective partial differentiations of r_b . In the transformed coordinate system, U , V , and W are given by

$$U = \cos \alpha + \phi_\xi - r_{bz} B(1 - \zeta)^2 \phi_\zeta \quad (9a)$$

$$V = \cos \Omega \sin \alpha + B(1 - \zeta)^2 \phi_\zeta \quad (9b)$$

$$W = (1/r)[\phi_\Omega - r_{b\omega} B(1 - \zeta)^2 \phi_\zeta] \\ - \sin \Omega \sin \alpha \quad (9c)$$

The boundary condition for the flow can be simply stated as $\bar{V} \cdot \bar{n} = 0$ on the body surface where $\bar{n}$ is the unit outward normal. Referring to Fig. 3, this condition of tangency on the surface can be further developed as follows:

The unit normal $\bar{n}$ can be expressed as

$$\bar{n} = -\cos \beta' \sin \theta \bar{e}_z + \cos \beta' \cos \theta \bar{e}_r + \sin \beta' \bar{e}_\omega$$

where $\tan \theta = (\partial r_b / \partial z)$. The boundary condition thus can be expressed as

$$V = U \tan \theta - W \tan \beta' / \cos \theta \text{ at } \zeta = 0$$

since

$$(\tan \beta' / \cos \theta) = \tan \beta = -(1/r_b) (\partial r_b / \partial \omega) = -(1/r_b) r_{b\omega}$$

and upon substituting Eq. (9) into the above expression, the boundary condition becomes

$$\begin{aligned} \phi_\zeta(0) = & [(\phi_\xi(0) + \cos \alpha) r_{bz} + (\phi_\Omega(0)/r_b - \sin \Omega \sin \alpha) \\ & \cdot (1/r_b) r_{b\omega} - \cos \Omega \sin \alpha] / \{B[1 + r_{bz}^2 + (1/r_b^2) r_{b\omega}^2]\} \end{aligned} \quad (10)$$

where (0) signifies that the quantities are to be evaluated at $\zeta = 0$.

The boundary condition far away from the body now becomes

$$\phi = 0 \text{ at } \zeta = 1 \text{ or } \xi = \pm\infty \quad (11)$$

However, in view of the experience gained from previous study [1] on the flow without angle of attack, it is appropriate to assume that $\phi = 0$ at $\xi = -2$ and $\xi = +3$ where the length, L , of the body (Fig. 1) has been chosen as unity.

Equation (7) may now be solved by finite difference calculations through relaxational method. However, as mentioned in INTRODUCTION, Eq. (7) cannot be applied for the points along the axis ahead of the body. The governing equation in the Cartesian system of coordinates must be introduced for all the points on the axis. This equation is given by

$$\begin{aligned} & [1 - (\phi_z^2/c^2)] \phi_{zz} + [1 - (\phi_x^2/c^2)] \phi_{xx} + [1 - (\phi_y^2/c^2)] \\ & \cdot \phi_{yy} - [(2\phi_x \phi_z)/c^2] \phi_{xz} - [(2\phi_x \phi_y)/c^2] \phi_{xy} \\ & - [(2\phi_y \phi_z)/c^2] \phi_{yz} = 0 \end{aligned} \quad (12)$$

Since this equation is only applied to points on the Z-axis ahead of the body and the fact that the flow is symmetric with respect to the x-z plane leads to $\phi_y = 0$ for all points on the axis, Eq. (12) is immediately simplified to

$$[1 - (\phi_z^2/c^2)] \phi_{zz} + [1 - (\phi_x^2/c^2)] \phi_{xx} + \phi_{yy} - [(2\phi_x \phi_z)/c^2] \phi_{xz} = 0 \quad (13)$$

The disturbed potential function, ϕ , is introduced according to $\phi = V_\infty(z \cos \alpha + x \sin \alpha + \phi)$ which is precisely the same disturbed potential function ϕ introduced previously.

Equation (13) becomes

$$\{1 - [(\cos \alpha + \phi_z)^2/C^2]\} \phi_{zz} + \{1 - [(\sin \alpha + \phi_x)^2/C^2]\} \phi_{xx} + \phi_{yy} - \{[2(\cos \alpha + \phi_z)(\sin \alpha + \phi_x)]/C^2\} \phi_{xz} = 0 \quad (14)$$

where

$$C^2 = (1/M_\infty^2) + [(\gamma - 1)/2] [1 - (\cos \alpha + \phi_z)^2 - (\sin \alpha + \phi_x)^2]$$

When Eq. (14) is written in finite difference form with the central difference scheme, the grid points involved for a typical point on the z axis are shown in Fig. 4. It is recognized that all grid points are already the points needed in the finite difference formulations of Eq. (7) in the transformed system of coordinates (ξ, ζ, Ω) . No additional new grid points are introduced for the calculation of Eq. (14).

Finite Difference Equations

The computational space of $-2 \leq \xi \leq +3$, $0 \leq \zeta \leq 1$, and $0 \leq \Omega \leq \pi$ was discretized into a grid system where uniform values of $\Delta\zeta = 0.04$ and $\Delta\Omega = 30^\circ = 0.523$ rad were employed. Thus, calculations on seven meridional planes have been carried out. Since the present investigation has the main interest in the flow properties within the boattailed after-body region, variable values of $\Delta\xi$ were used. The grid system is shown in Fig. 5.

Many of the finite difference forms of the terms in Eq. (7) are different for locally subsonic or supersonic flow. They are given by

$$\begin{aligned}\phi_\xi &= \{\Delta\xi_1/[\Delta\xi_2(\Delta\xi_1 + \Delta\xi_2)]\} \phi_{i+1,j}^k \\ &\quad + [(\Delta\xi_2 - \Delta\xi_1)/(\Delta\xi_1 \Delta\xi_2)] \phi_{i,j}^k \\ &\quad - \{\Delta\xi_2/[\Delta\xi_1(\Delta\xi_1 + \Delta\xi_2)]\} \phi_{i-1,j}^k \\ &= AB1 \phi_{i+1,j}^k + AB2 \phi_{i,j}^k + AB3 \phi_{i-1,j}^k\end{aligned}\quad (15a)$$

$$\begin{aligned}\phi_{\xi\xi} &= \{2/[\Delta\xi_2(\Delta\xi_1 + \Delta\xi_2)]\} \phi_{i+1,j}^k - \{2/(\Delta\xi_1 \Delta\xi_2)\} \phi_{i,j}^k \\ &\quad + \{2/[\Delta\xi_1(\Delta\xi_1 + \Delta\xi_2)]\} \phi_{i-1,j}^k \\ &= BB1 \phi_{i+1,j}^k + BB2 \phi_{i,j}^k + BB3 \phi_{i-1,j}^k\end{aligned}\quad (15b)$$

$$\begin{aligned}\phi_{\xi\zeta} = & (1/2\Delta\zeta) [AB1(\phi_{i+1,j}^{k+1} - \phi_{i+1,j}^{k-1}) \\ & + AB2(\phi_{i,j}^{k+1} - \phi_{i,j}^{k-1}) + AB3(\phi_{i-1,j}^{k+1} - \phi_{i-1,j}^{k-1})] \quad (15c)\end{aligned}$$

$$\begin{aligned}\phi_{\xi\Omega} = & (1/2\Delta\Omega) [AB1(\phi_{i+1,j+1}^k - \phi_{i+1,j-1}^k) \\ & + AB2(\phi_{i,j+1}^k - \phi_{i,j-1}^k) \\ & + AB3(\phi_{i-1,j+1}^k - \phi_{i-1,j-1}^k)] \quad (15d)\end{aligned}$$

for locally subsonic flow, and

$$\begin{aligned}\phi_{\xi} = & \{[\Delta\xi_0 + 2\Delta\xi_1]/[\Delta\xi_1(\Delta\xi_0 + \Delta\xi_1)]\} \phi_{i,j}^k \\ & - [(\Delta\xi_1 + \Delta\xi_0)/(\Delta\xi_1 \Delta\xi_0)] \phi_{i-1,j}^k \\ & + \{[\Delta\xi_1]/[\Delta\xi_0(\Delta\xi_0 + \Delta\xi_1)]\} \phi_{i-2,j}^k \\ = & AP1 \phi_{i,j}^k + AP2 \phi_{i-1,j}^k + AP3 \phi_{i-2,j}^k \quad (16a)\end{aligned}$$

$$\begin{aligned}\phi_{\xi\xi} = & \{2/[\Delta\xi_1(\Delta\xi_0 + \Delta\xi_1)]\} \phi_{i,j}^k \\ & - [2/(\Delta\xi_0\Delta\xi_1)] \phi_{i-1,j}^k \\ & + \{2/[\Delta\xi_0(\Delta\xi_0 + \Delta\xi_1)]\} \phi_{i-2,j}^k \\ = & BP1 \phi_{i,j}^k + BP2 \phi_{i-1,j}^k + BP3 \phi_{i-2,j}^k \quad (16b)\end{aligned}$$

$$\begin{aligned}\phi_{\xi\zeta} = & (1/2\Delta\zeta) [AP1(\phi_{i,j}^{k+1} - \phi_{i,j}^{k-1}) + AP2(\phi_{i-1,j}^{k+1} - \phi_{i-1,j}^{k-1}) \\ & + AP3(\phi_{i-2,j}^{k+1} - \phi_{i-2,j}^{k-1})] \quad (16c)\end{aligned}$$

$$\begin{aligned}\phi_{\xi\Omega} = & (1/2\Delta\Omega) [AP1(\phi_{i,j+1}^k - \phi_{i,j-1}^k) \\ & + AP2(\phi_{i-1,j+1}^k - \phi_{i-1,j-1}^k) \\ & + AP3(\phi_{i-2,j+1}^k - \phi_{i-2,j-1}^k)] \quad (16d)\end{aligned}$$

for locally supersonic flow where

$$\Delta\xi_2 = \xi_{i+1} - \xi_i; \Delta\xi_1 = \xi_i - \xi_{i-1}; \text{ and}$$

$$\Delta\xi_0 = \xi_{i-1} - \xi_{i-2} \quad (17)$$

Other finite difference forms are

$$\phi_{\zeta} = [(\phi_{i,j}^{k+1} - \phi_{i,j}^{k-1})]/2 \Delta\zeta \quad (18a)$$

$$\phi_{\zeta\zeta} = (\phi_{i,j}^{k+1} - 2\phi_{i,j}^k + \phi_{i,j}^{k-1})/(\Delta\zeta)^2 \quad (18b)$$

$$\phi_{\Omega} = (\phi_{i,j+1}^k - \phi_{i,j-1}^k)/2\Delta\Omega \quad (18c)$$

$$\phi_{\Omega\Omega} = (\phi_{i,j+1}^k - 2\phi_{i,j}^k - \phi_{i,j-1}^k) / [(\Delta\Omega)^2] \quad (18d)$$

$$\phi_{\Omega\zeta} = [(\phi_{i,j+1}^{k+1} - \phi_{i,j+1}^{k-1}) - (\phi_{i,j-1}^{k+1} - \phi_{i,j-1}^{k-1})] / [4\Delta\zeta \Delta\Omega] \quad (18e)$$

Equation (7) may now be written in finite difference form for the point $(\xi_i, \omega_j, \zeta_k)$ and the resulting expression, when arranged in tridiagonal form for all points along the ζ coordinate with fixed values of ξ_i, ω_j , is

$$\begin{aligned} & [B^2(1-\zeta)^4 \cdot EE + (\Delta\zeta/2) B(1-\zeta)^2 \cdot DE \cdot AB2] \\ & \cdot \phi_{i,j}^{k+1} + [\Delta\zeta^2 \cdot DD \cdot BB2 - 2B^2(1-\zeta)^4 \cdot EE \\ & - 2(\Delta\zeta/\Delta\Omega)^2 \cdot FF] \phi_{i,j}^k + [B^2(1-\zeta)^4 \cdot EE \\ & - (\Delta\zeta/2) B(1-\zeta)^2 \cdot DE \cdot AB2] \phi_{i,j}^{k-1} = \\ & - (\Delta\zeta/2) B(1-\zeta)^2 \cdot DE \cdot [AB1(\phi_{i+1,j}^{k+1} - \phi_{i+1,j}^{k-1}) \\ & + AB3(\phi_{i-1,j}^{k+1} - \phi_{i-1,j}^{k-1})] - (\Delta\zeta)^2 \\ & \cdot [DD \cdot (BB1 \phi_{i+1,j}^k + BB3 \phi_{i-1,j}^k) + [FF/(\Delta\Omega)^2] \\ & \cdot (\phi_{i,j+1}^k + \phi_{i,j-1}^k) + H] \quad (19) \end{aligned}$$

for subsonic flow, and

$$\begin{aligned}
 & [B^2(1 - \zeta)^4 \cdot EE + (\Delta\zeta/2) B(1 - \zeta)^2 \cdot DE \cdot AP1] \\
 & \cdot \phi_{i,j}^{k+1} + [(\Delta\zeta)^2 \cdot DD \cdot BP1 - 2B^2(1 - \zeta)^4 \cdot EE \\
 & - 2(\Delta\zeta/\Delta\Omega)^2 \cdot FF] \phi_{i,j}^k + [B^2(1 - \zeta)^4 \cdot EE \\
 & - (\Delta\zeta/2) B(1 - \zeta)^2 \cdot DE \cdot AP1] \phi_{i,j}^{k-1} = \\
 & - (\Delta\zeta/2) B(1 - \zeta)^2 \cdot DE \cdot [AP2(\phi_{i-1,j}^{k+1} - \\
 & \phi_{i-1,j}^{k-1}) + AP3(\phi_{i-2,j}^{k+1} - \phi_{i-2,j}^{k-1})] \\
 & - (\Delta\zeta)^2 [DD \cdot (BP2 \phi_{i-1,j}^k + BP3 \phi_{i-2,j}^k) \\
 & + [FF/(\Delta\Omega)^2] (\phi_{i,j+1}^k + \phi_{i,j-1}^k) + H] \quad (20)
 \end{aligned}$$

for supersonic flow where

$$\begin{aligned}
 H = & B(1 - \zeta)^2 \cdot EF \cdot \phi_{\Omega\zeta} + FD \cdot \phi_{\Omega\xi} \\
 + & B(1 - \zeta)^2 [E - 2B(1 - \zeta) \cdot EE] \phi_{\zeta} + G \quad (21)
 \end{aligned}$$

To account for the boundary condition on the body surface ($\zeta = 0, k = 1$), $\phi_{\zeta}(0)$ was evaluated for the points on the surface according to Eq. (10) and a row of grid points was introduced at $\zeta = -\Delta\zeta$ in every meridional plane whose values of ϕ (identified as $\phi_{i,j}^b$) were given by

$$\phi_{i,j}^b = \phi_{i,j}^{k=2} - 2\Delta\zeta \phi_{\zeta}(\xi_i, \omega_j, \zeta = 0) \quad (22)$$

Similarly, due to symmetry, ϕ_{Ω} and $r_{b\omega}$ vanish for all grid points on the meridional planes of $\omega = 0$ ($j = 1$) and $\omega = \pi$ ($j = 7$). Again, a row of grid points was introduced for each ζ value at $\omega = -\Delta\omega$ or $\omega = \pi + \Delta\omega$ whose values of ϕ (identified as $\phi_{i,b}^k$) were given by

$$\phi_{i,b}^k = \phi_{i,j=2}^k \text{ for } \omega = 0 \quad (23a)$$

$$\phi_{i,b}^k = \phi_{i,j=6}^k \text{ for } \omega = \pi \quad (23b)$$

When Eq. (19) or (20) is written for all points along the ζ coordinate (for fixed ξ_i, ω_j values), the coefficient matrix for ϕ is tridiagonal, and values of ϕ can be solved by standard and efficient methods. Line relaxational calculations can be carried out along each plane of fixed ξ value in the direction of increasing ω and then sweeping from upstream toward downstream positions, and the information of the fictitious row of grid points created to account for the boundary condition was updated as soon as the calculations for that column of grid points were completed. However, before carrying out these calculations, finite difference calculations must be performed for the points along the z -axis ahead of the body.

Referring to Fig. 4, the finite difference forms of the terms in Eq. (14) are given by*

*The finite difference forms are for subsonic flow ahead of the body. For supersonic flows, expressions for ϕ_z , ϕ_{zz} , and ϕ_{zx} must be modified.

$$\phi_z = AB1 \phi_{i+1}^p + AB2 \phi_i^p + AB3 \phi_{i-1}^p \quad (24a)$$

$$\phi_{zz} = BB1 \phi_{i+1}^p + BB2 \phi_i^p + BB3 \phi_{i-1}^p \quad (24b)$$

$$\begin{aligned} \phi_{zx} = [1/(2\Delta\eta_1)] [AB1(\phi_{i+1}^n - \phi_{i+1}^s) \\ + AB2(\phi_i^n - \phi_i^s) + AB3(\phi_{i-1}^n - \phi_{i-1}^s)] \end{aligned} \quad (24c)$$

$$\phi_x = (\phi_i^n - \phi_i^s)/2\Delta\eta_1 \quad (24d)$$

$$\phi_{xx} = (\phi_i^n - 2\phi_i^p + \phi_i^s)/[(\Delta\eta_1)^2] \quad (24e)$$

$$\phi_{yy} = [2(\phi_i^e - \phi_i^p)]/[(\Delta\eta_1)^2] \quad (24f)$$

where $\Delta\eta_1$ corresponds to $\Delta\zeta$ between the first and the second grid points along the ζ coordinate and superscripts p,n,s, and e refer to the points on the axis, north, south, and east of the axis point, respectively. Upon writing Eq. (14) in finite difference form, the value of ϕ_i^p can be readily obtained from

$$\begin{aligned}
\phi_i^p = & [(\Delta\eta_1)^2 \{ [2(\cos \alpha + \phi_z)(\sin \alpha + \phi_x)/C^2] \phi_{zx} \\
& - [1 - (\cos \alpha + \phi_z)^2/C^2] [BB1 \phi_{i+1}^p + BB3 \phi_{i-1}^p] \} \\
& - [1 - (\sin \alpha + \phi_x)^2/C^2] (\phi_i^n + \phi_i^s) - 2\phi_i^e \} / \\
& [(\Delta\eta_1)^2 BB2 [1 - (\cos \alpha + \phi_z)^2/C^2] \\
& - 2[1 - (\sin \alpha + \phi_x)^2/C^2] - 2] \quad (25)
\end{aligned}$$

Viscous Flow

The quasi-axisymmetric boundary layer calculations are based on the integral method of Sasman and Cresci [4] and follow in the same formulations as described in [1]. The pair of differential equations for the momentum thickness θ_c and the form factor H_c are integrated numerically with the given inviscid surface flow condition along the axial direction as the guiding free stream. These equations are presented again as follows:

$$\begin{aligned}
(df/dx) = & 1.268 [(-f/M_e)(dM_e/dx)(1 + g_w H_i) \\
& - (f/r_b)(dr_b/dx) + A] \quad (26)
\end{aligned}$$

and

$$\begin{aligned}
dH_i/dx = & -(1/2M_e) (dM_e/dx) [H_i (H_i + 1)^2 (H_i - 1)] \\
& \cdot \{1 + (g_w - 1) [(H_i^2 + 4H_i - 1)] / [(H_i + 1)(H_i + 3)]\} \\
& + [(H_i^2 - 1)/f] A [H_i - \{0.011(H_i + 1)(H_i - 1)^2 / \\
& H_i^2\} \cdot (2/c_f) (T_{0,e}/\bar{T})] \quad (27)
\end{aligned}$$

where

$$f = [(M_e c_o \theta_i)/v_o]^{1.268}; \quad (28a)$$

$$\theta_i = \theta(T_e/T_{0,e})^{(\gamma+1)/[2(\gamma-1)]} \quad (28b)$$

$$g_w \equiv T_w/T_{0,e} \quad (28c)$$

$$A = 0.123 e^{-1.561 H_i}$$

$$\cdot [(M_e c_o)/v_o] [T_e/\bar{T}] [T_e/T_{0,e}]^3 [\bar{\mu}/\mu_o]^{0.268} \quad (28d)$$

$$\begin{aligned}
\bar{T}/T_{0,e} = & 0.5 (T_w/T_{0,e}) + 0.22 Pr^{1/3} \\
& + (0.5 - 0.22 Pr^{1/3}) (T_e/T_{0,e}) \quad (28e)
\end{aligned}$$

$$\begin{aligned}
H_c = & H_i g_w [1 + [(\gamma - 1)/2] M_e^2] \\
& + [(\gamma - 1)/2] M_e^2 \quad (28f)
\end{aligned}$$

$$(c_f/2) = 0.123 e^{-1.561 H_i} [M_e c_o \theta_i / v_o]^{-0.268} (T_e/\bar{T}) (\bar{\mu}/\mu_o)^{0.268}$$

and x is the length along the surface of the body. (28g)

METHOD OF CALCULATIONS

The method of calculations follows in the same manner as that described previously for cases with zero angle of attack [1]. The successive refinement of grid spacing in the ξ -direction is again employed and its number of grid points changes from 26 to 51 and finally to 101. Initially, calculations are carried out for all grid points of the same ξ location (they are all on the plane perpendicular to the z axis) by performing line relaxations from $\omega = 0$ to $\omega = \pi$, followed by all grid points in the next downstream constant ξ -plane, and so on. The information at the grid point created to account for the boundary condition on the surface is updated, according to Eq. (22) immediately after the calculations for that column of grid points are completed. As the end of a complete cycle of relaxation for all ξ stations, values of ϕ for all points on the axis ahead of the body are updated according to Eq. (25). From the experience gained in the study of cases with zero angle of attack, frequent quasi-axisymmetric boundary layer calculations are not required. These calculations were carried out at the end of second, fifth, ninth, fourteenth, twentieth, ..., cycle of inviscid flows calculations and the corrections to the inviscid body configuration through the displacement effect of the viscous layer were applied accordingly (see Ref. [1] for details of this correction). After the successive grid refinements, the final flow pattern is eventually established when the successive change of ϕ is less than a small number ($\epsilon = 3 \times 10^{-5}$)* for all grid points throughout the field.

*To save computational time, this number is ten times larger than the value employed for cases in Ref. [1].

RESULTS OF CALCULATIONS

After the completion of developing the computer program for flows with angle of attack, one possible way of checking the program is to reproduce the results when the angle of attack is set to zero. Indeed, the results obtained for different meridional planes at $M_\infty = 0.8$ are agreeable to each other and are shown in Fig. 6 where the previous results with $\epsilon = 3 \times 10^{-6}$ (Ref. [1]) and the corresponding experimental data are also presented for comparison. The slight difference between the two calculations appears to come from the difference in ϵ for the criterion of convergence.

Calculations for the case of flow with angle of attack with $\alpha = 4^\circ$ at $M_\infty = 0.8$ have also been carried out. The inviscid and viscous flow results are presented in Fig. 7. Inviscid flow results were obtained by bypassing the boundary layer calculations. It is obvious that the viscous flow modifies the pressure distribution on the boattailed afterbody so much that it is impossible to obtain reasonable results without consideration of its effects. The experimental results obtained by Shrewsbury [5] corresponding to this flow case are presented in Fig. 8 along with the viscous flow results for comparison purposes. Although the viscous layer is calculated from the quasi-axisymmetric approach, the final predicted level of the pressure coefficient, as well as its trend of variation is reasonable in comparison with experimental data. It is believed, however, that the quasi-axisymmetric boundary layer calculations must

be replaced by the three-dimensional boundary layer calculations before excellent agreement can be achieved. The corresponding displacement thickness resulting from the quasi-axisymmetric boundary layer calculations are shown in Fig. 9. The computational time required for this case is 225 seconds on the CDC CYBER 175 computer system. Since the computational cost is expensive while the method of viscous flow analysis is not accurate, no extensive calculations have been performed.

CONCLUSION

A computer program dealing with an axisymmetric body at angle of attack within the transonic flow regime has been developed. The viscous layer growth on the body introduces important modifications to the inviscid body geometry within this flow regime. A quasi-axisymmetric boundary layer estimation serves to show the correct trends of the pressure distribution but would not produce accurate results when compared with the experimental data. A fully three-dimensional compressible turbulent boundary layer treatment must be implemented to improve this situation. Nevertheless, the investigation does establish the fact that inviscid flow field of an axisymmetric body immersed in a transonic flow at angle of attack can be successfully calculated.

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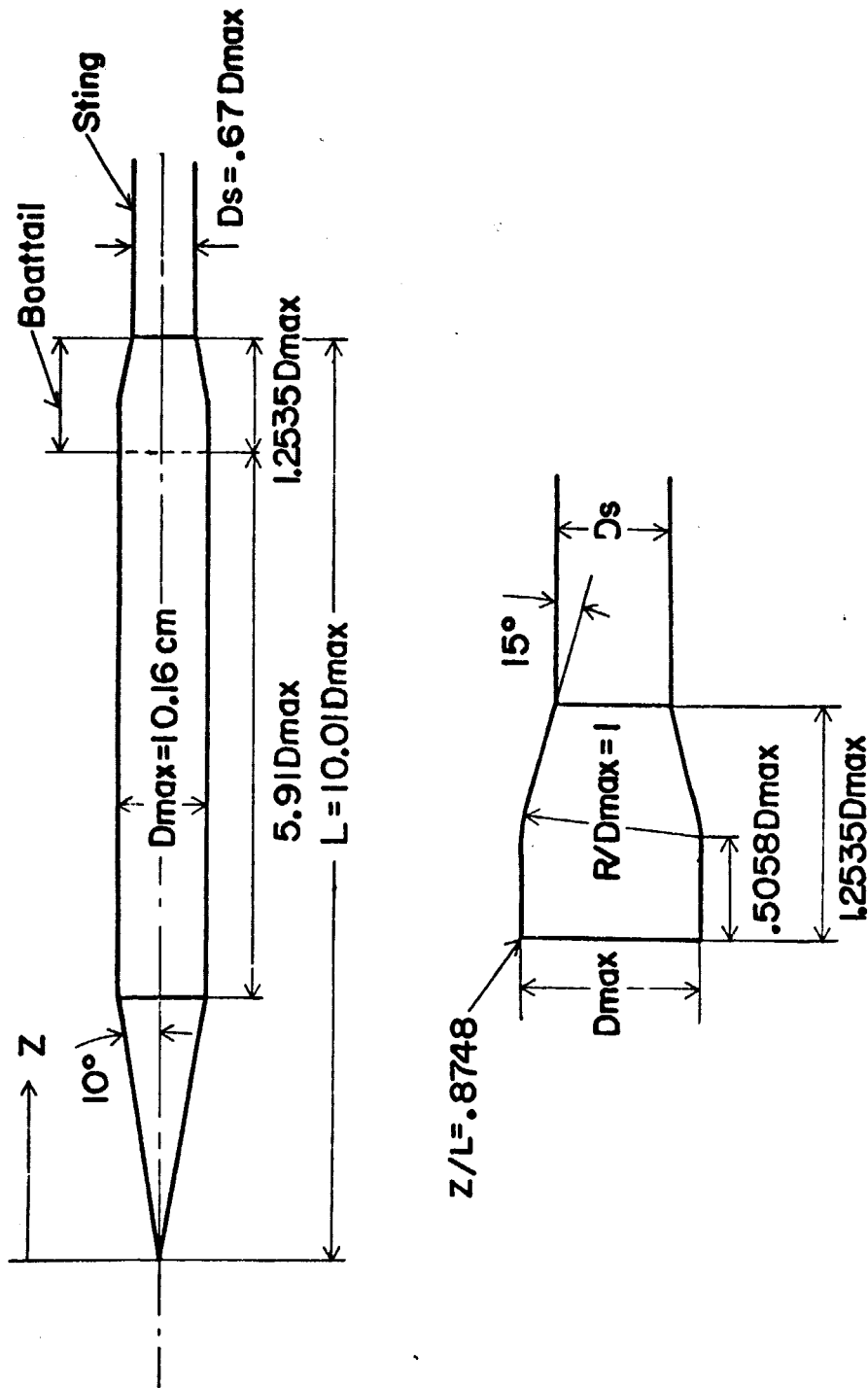


Figure 1 Geometrical Configuration of the Model for Numerical Calculations

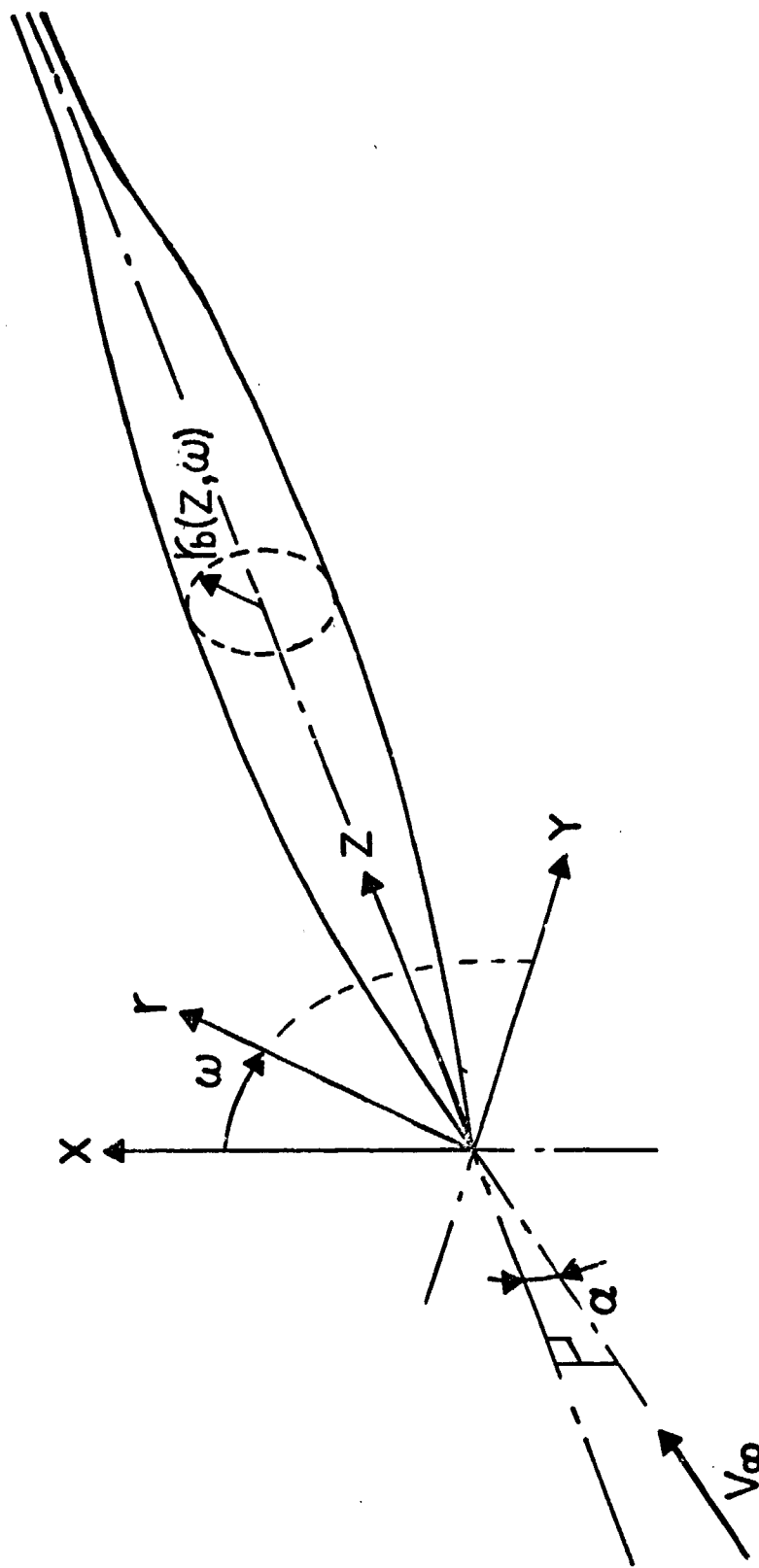


Figure 2 Transonic Flow past a Body at an Angle of Attack

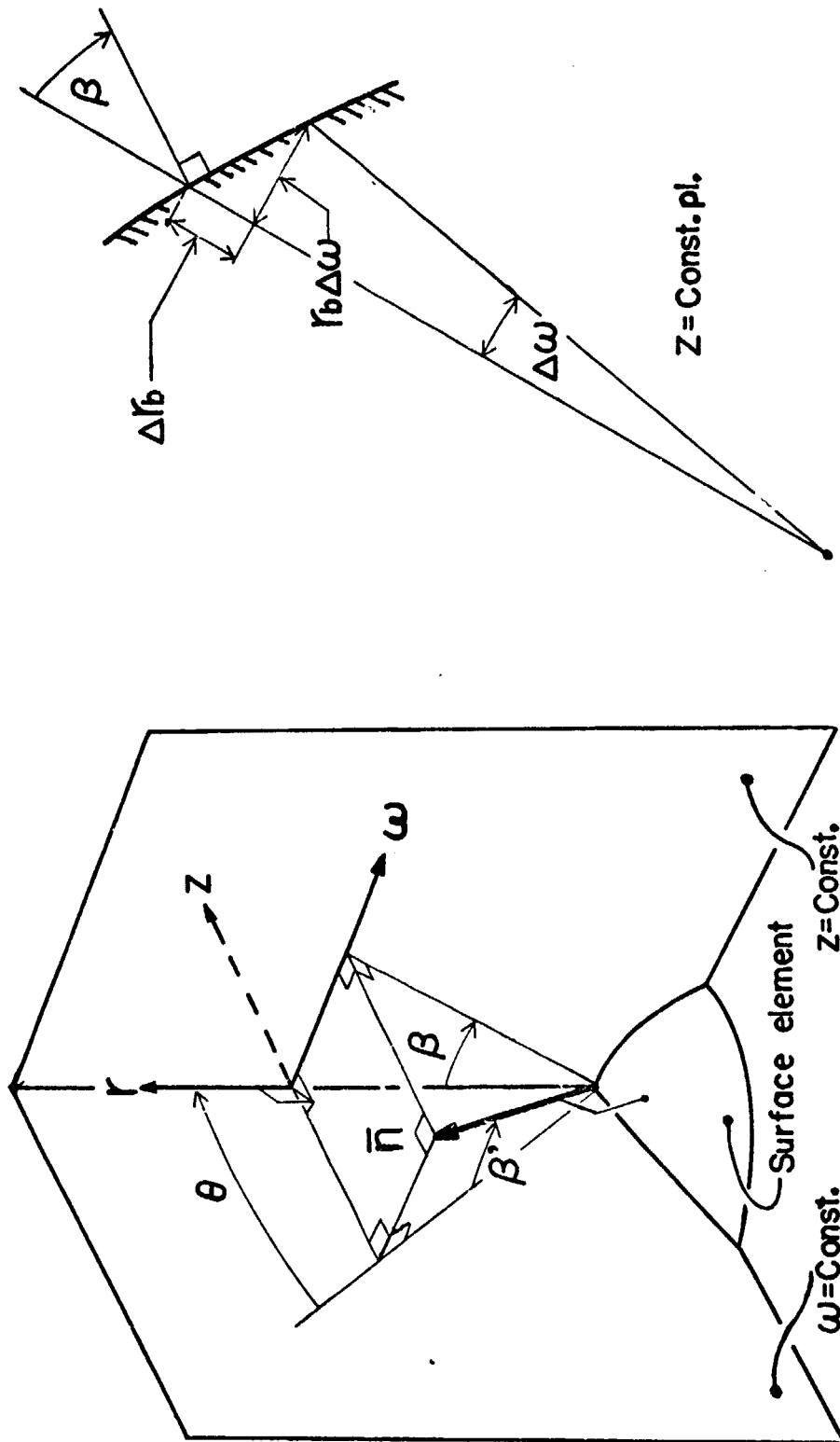


Figure 3 The Geometric Diagram for Boundary Conditions on the Surface

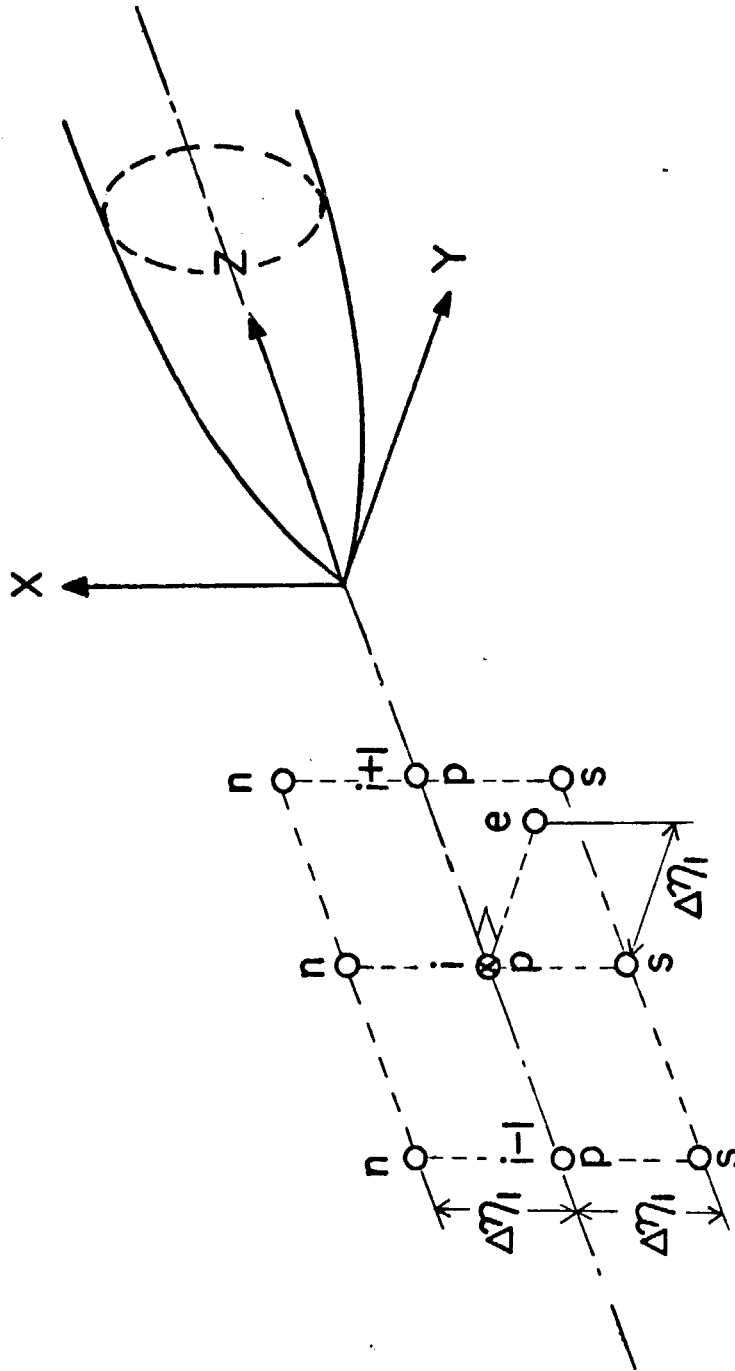


Figure 4 Grid Points on the Axis ahead of the Body

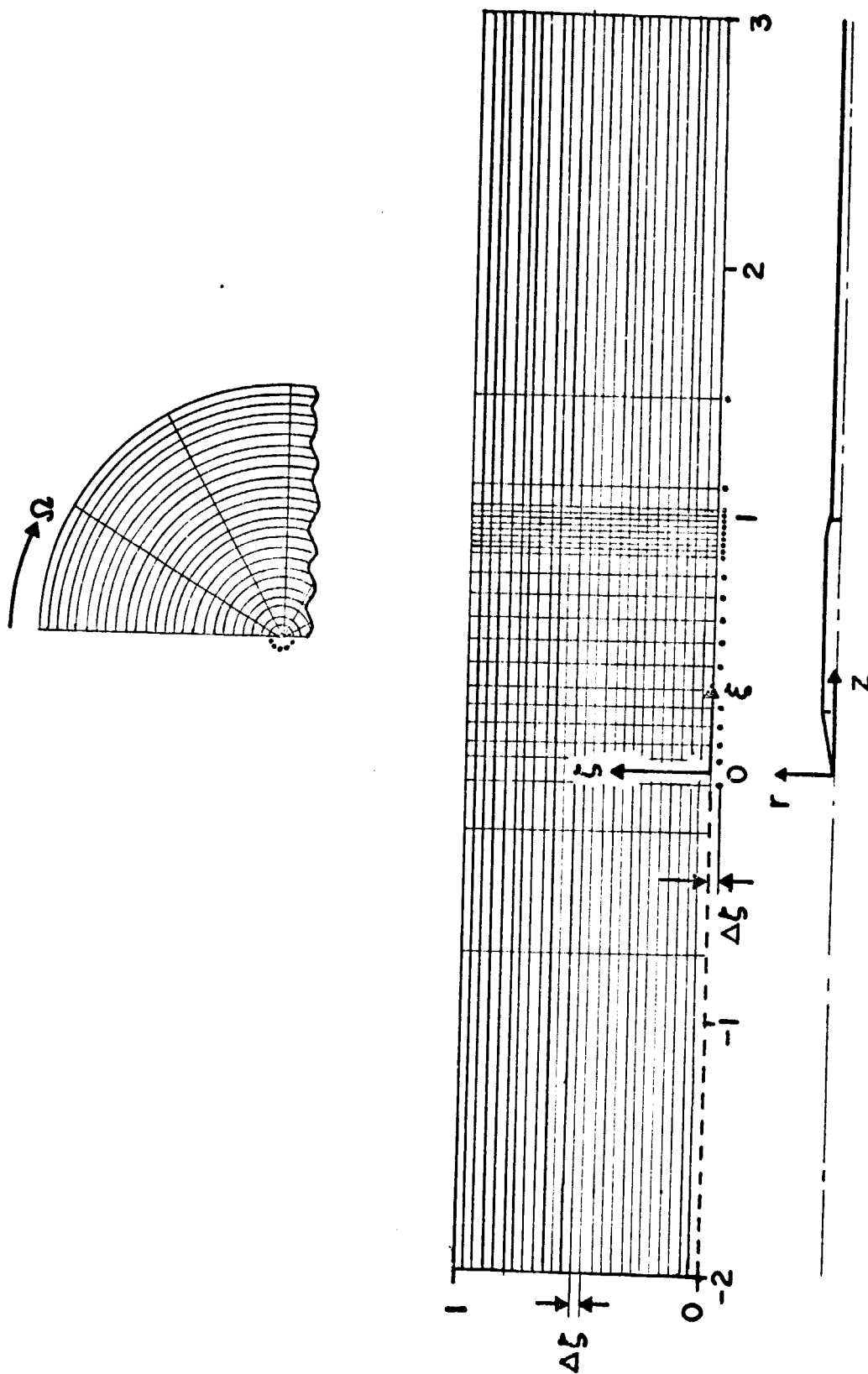


Figure 5 Computational Plane with Grid Layout with Respect to the Body

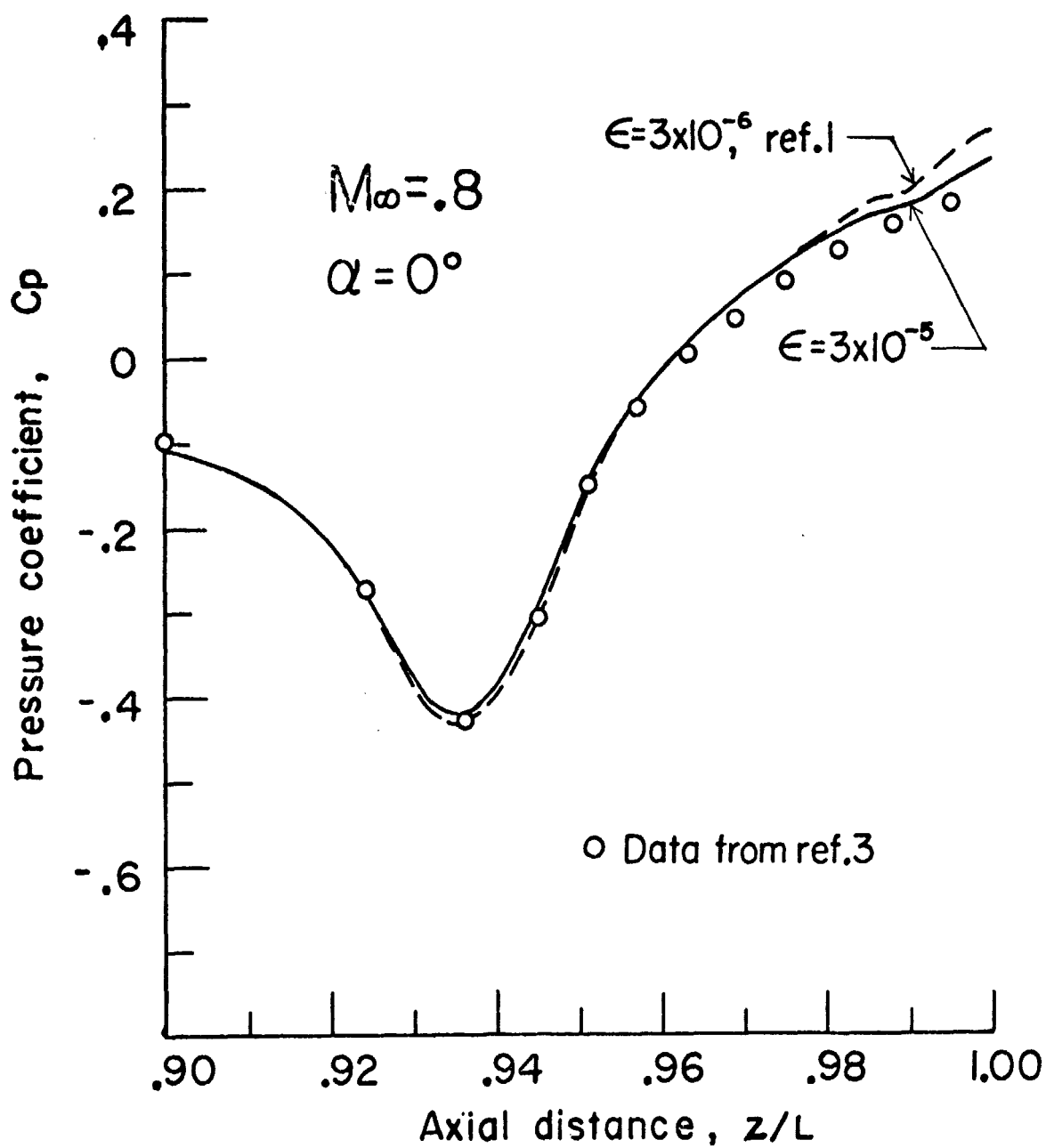


Figure 6 Results for Zero Angle of Attach ($M_\infty = 0.8$, $\alpha = 0$)

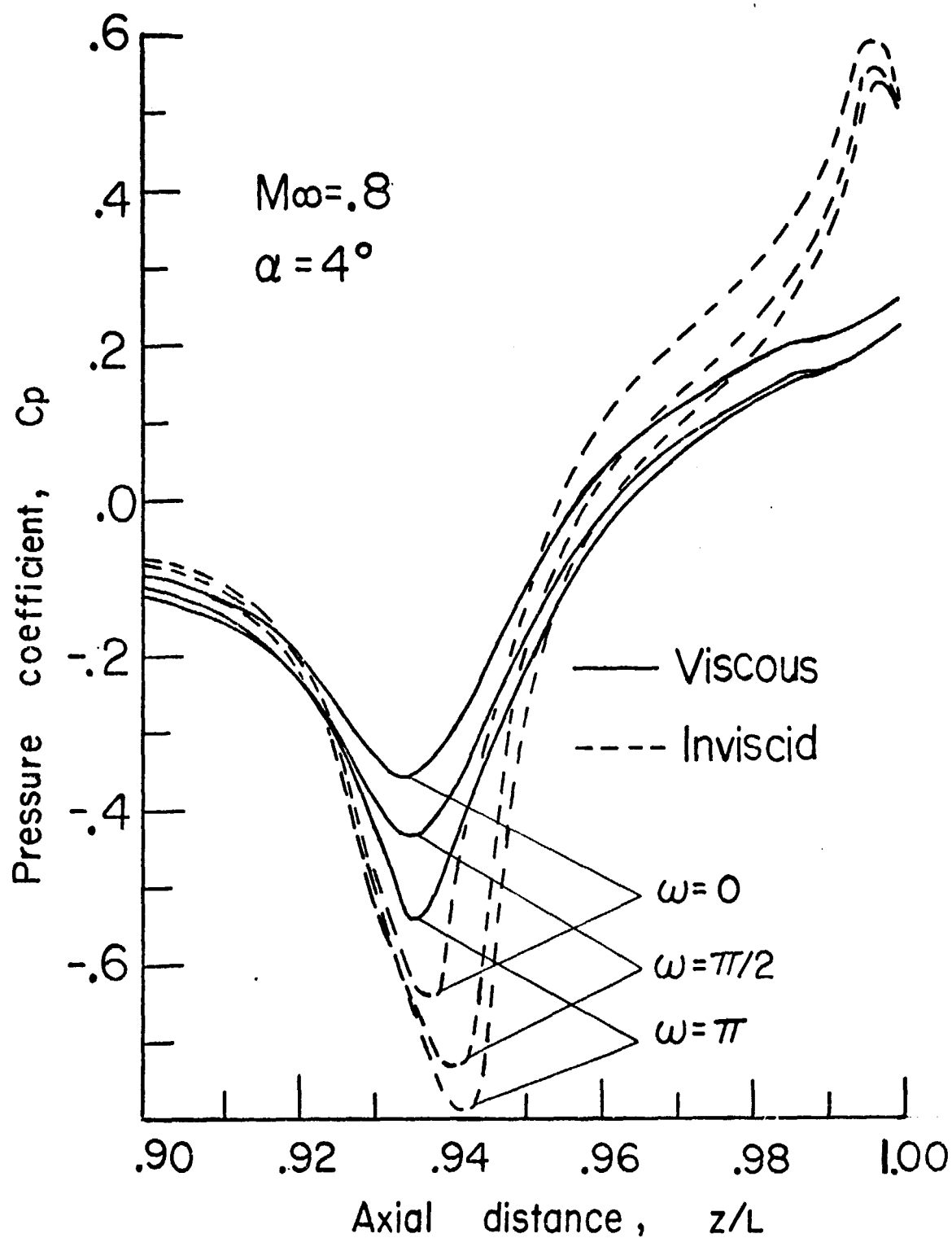


Figure 7 Comparison of Inviscid and Viscous Flow Results

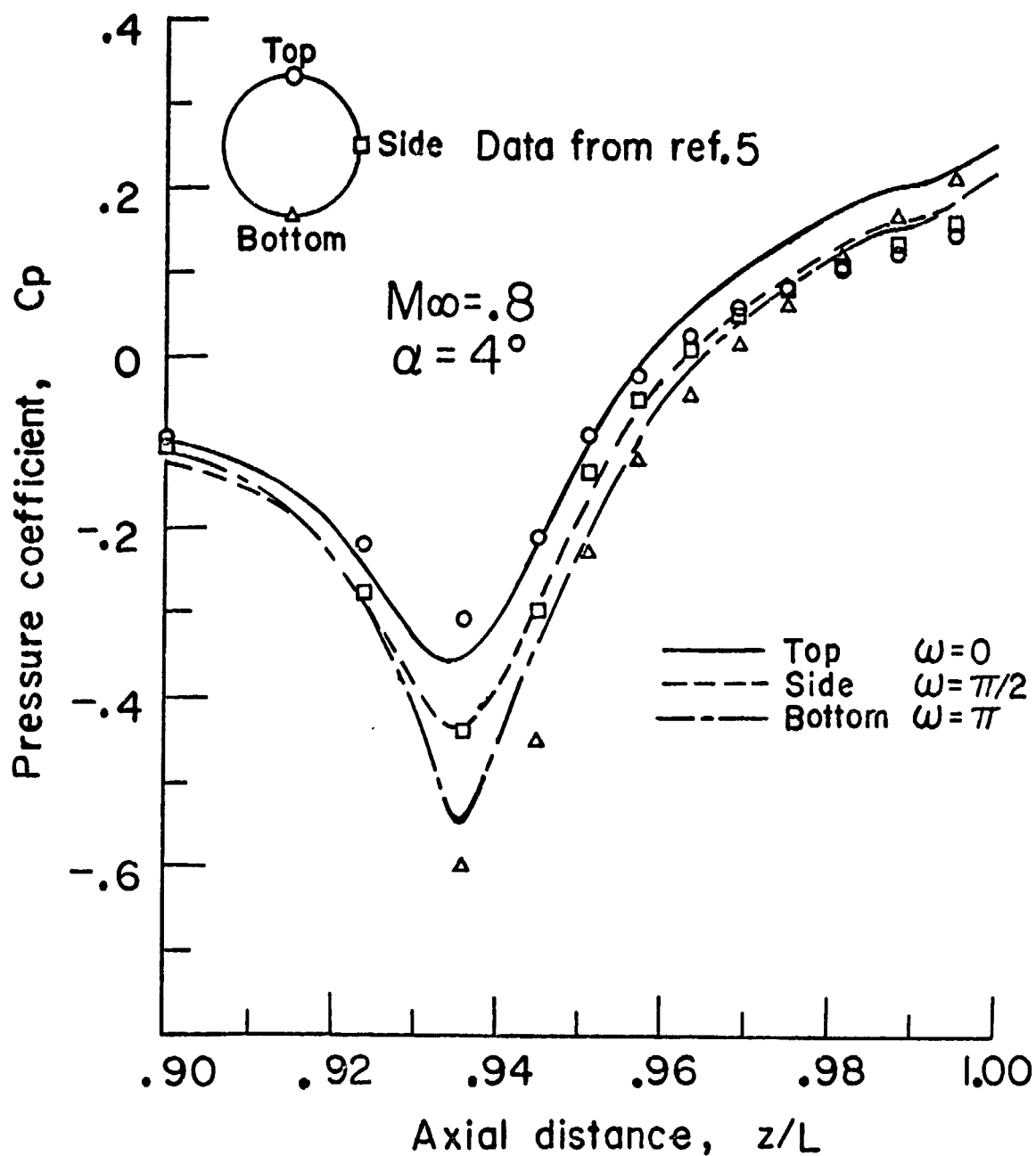


Figure 8 Comparison of Numerical Results with the Experimental Data

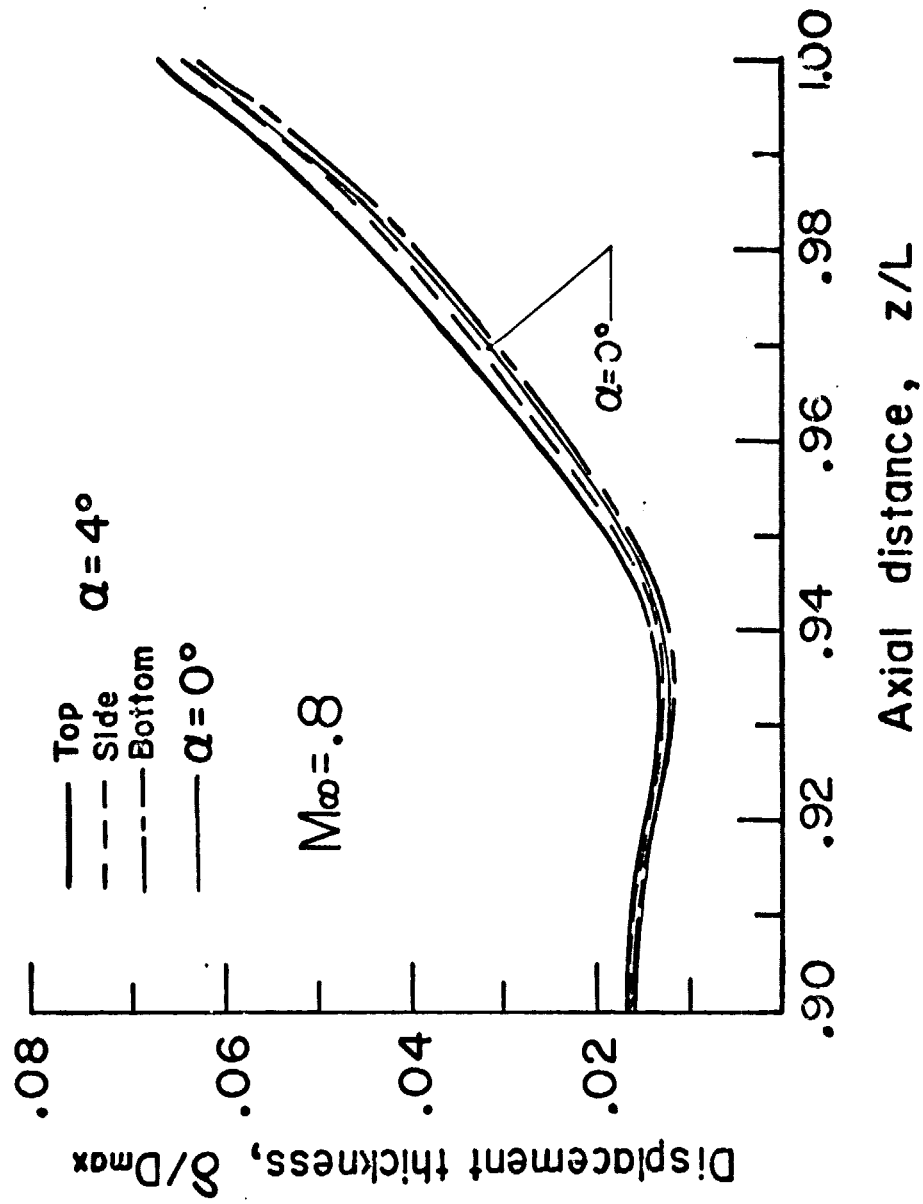


Figure 9 The Growth of the Displacement Thickness of the Boundary Layer

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NOMENCLATURE

A, f	functions for boundary layer calculations, defined in Eqs. (26) and (28)
AB1, AB2, AB3	coefficient functions at point where flow is subsonic, defined in Eq. (15a)
AP1, AP2, AP3	coefficient functions at point where flow is supersonic, defined in Eq. (16a)
B	radial stretch parameter ($B = 8$)
BB1, BB2, BB3	coefficient functions at point where flow is subsonic, defined in Eq. (15b)
BP1, BP2, BP3	coefficient functions at point where flow is supersonic, defined in Eq. (16b)
C	dimensionless speed of sound
C_p	pressure coefficient $(p - p_\infty)/(1/2) \rho_\infty V_\infty^2$
c	local speed of sound
c_f	skin friction coefficient
DD, EE, DE, FF, EF, FD, E, G, H	coefficients in inviscid analysis, defined Eqs. (8) and (21)
$D_{\max}$	maximum model diameter
D_s	sting diameter
H_c	shape factor of compressible boundary layer
H_i	shape factor of incompressible boundary layer
i, j, k	indices for finite different formulations
L	body length to end of boattail
M	Mach number
Pr	Prandtl number
p	pressure
R	boattail juncture radius of curvature

r	radial distance from the axis
$r_b(z, \omega)$	local body radius
T	temperature
$\bar{T}$	reference temperature for the boundary-layer calculation
U, V, W	dimensionless velocity components in axial direction, radial and meridional directions, respectively
u, v, w	velocity components in axial, radial, and meridional directions
V_∞	free stream velocity
x	distance along body surface, or Cartesian coordinate
y, z	Cartesian coordinates
γ	ratio of specific heats
δ^*	displacement thickness of the boundary layer
$\Delta\zeta$	uniform spacing of the finite grid system in the ζ -direction
$\Delta\xi$	nonuniform spacing of the finite grid system in the ξ -direction
η	transformed coordinate, $r - r_b$
θ_c, θ_i	momentum thickness of the compressible and the corresponding incompressible boundary layers
μ	viscosity
$\bar{\mu}$	viscosity evaluated at $\bar{T}$ temperature
ν	kinematic viscosity
ξ	transformed axial coordinate
Φ	potential function
ϕ	normalized disturbance potential function
ω	meridional coordinate
Ω	transformed meridional coordinate
α	angle of attack
β, β'	coordinate angles, see Fig. 3

Subscripts

e	edge of boundary layer
r, z, ω	partial derivatives with respect to r, ω, z , respectively
w	wall
ξ	partial derivative with respect to ξ
0	stagnation state
∞	approaching free stream
x, y, z	partial derivatives with respect to $x, y, z, \zeta, \xi, \omega, \Omega$, respectively
$\zeta, \xi, \omega, \Omega$	

Superscripts

p, n, s, e	grid points on the axis ahead of the body north, south and east of this point
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